

Photon-number-resolving decoy state quantum key distribution

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Abstract

In this paper, a photon-number-resolving decoy state quantum key distribution scheme is presented based on recent experimental advancements. A new upper bound on the fraction of counts caused by multiphoton pulses is given. This upper bound is independent of intensity of the decoy source, so that both the signal pulses and the decoy pulses can be used to generate the raw key after verified the security of the communication. This upper bound is also the lower bound on the fraction of counts caused by multiphoton pulses as long as faint coherent sources and high lossy channels are used. We show that Eve's coherent multiphoton pulse (CMP) attack is more efficient than symmetric individual (SI) attack when quantum bit error rate is small, so that CMP attack should be considered to ensure the security of the final key. finally, optimal intensity of laser source is presented which provides 23.9 km increase in the transmission distance.

I. INTRODUCTION

Quantum key distribution (QKD) is a physically secure method, by which private key can be created between two partners, Alice and Bob, who share a quantum channel and a public authenticated channel [1]. The key bits then be used to implement a classical private key cryptosystem, or more precisely called *one – time pad* algorithm, to enable the partners to communicate securely. The best known QKD is the BB84 protocol published by Bennett and Brassward in 1984 [2], security of which has been studied deeply [3-7].

Experimental BB84 QKD was demonstrated by many groups [8]. An optical BB84 QKD system includes the photon sources, quantum channels, single-photon detectors, and quantum random-number generators. In principle, optical quantum cryptography is based on the use of single-photon Fock states. However, perfect single-photon sources are difficult to realize experimentally. Practical implementations rely on weak laser pulses in which photon number distribution obeys Possionian statistics. Thus, no-cloning principle is ineffective in the case of multiphoton pulses. If the quantum channel is high lossy, Eve can obtain full information of the final key by using photon number splitting (PNS) attack without being detected [9-13]. In GLLP [7], it has been shown that the secure final key of BB84 protocol can be extracted from sifted key at the asymptotic rate

$$R = (1 - \Delta) - H_2(e) - H_2(e + \Delta), \quad (1)$$

where e is the quantum bit error rate (QBER) found in the verification test and Δ is the fraction of counts caused by multiphoton pulses. This means that both the QBER e and the fraction of tagged signals Δ are important to generate the secure final key. It has been shown that Eve's PNS attack will be limited when Alice and Bob use the decoy-state protocols [14-20] or the nonorthogonal states scheme [21]. In the decoy-state protocols [14-20], an important assumption is that the detection apparatus cannot resolve the photon number of arriving signals. Recently, some photon-number-resolving detection apparatus were presented [22-24], especially the noise-free high-efficiency photon-number-resolving detectors [24]. Thus, a lower upper bound on the fraction of counts Δ is desired with the photon-number-resolving detectors. As a matter of fact, Eve's some other attacks, such as coherent multiphoton pulse (CMP) attack, should also be considered or else security of the final key will be unreliable.

In this paper, we present a photon-number-resolving decoy state (PDS) quantum key distribution scheme based on recent experimental advancements. We show that the upper bound on fraction of counts caused by multiphoton pulses is μ , no matter how high the channel loss is. We show that coherent multiphoton pulse (CMP) attack is more efficient than symmetric individual (SI) attack. We present the optimal approach to generate the sifted key from the raw key. Optimal parameter of intensity of laser source is presented to generate the secure final key. This paper is organized as follow: We first introduce our PDS QKD scheme. Then we discuss Eve’s CMP attack. Next, we present the optimal approach to generate the sifted key from the raw key. Then we discuss how to select optimal intensity of laser source to generate the secure final key. Finally, we discuss and conclude.

II. PHOTON-NUMBER-RESOLVING DECOY STATE QUANTUM KEY DISTRIBUTION

At present, practical “single-photon” sources rely on weak laser pulses in which photon number distribution obeys Possionian statistics. Most often, Alice sends to Bob a weak laser pulse in which she has encoded her bit. Each pulse is a priori in a coherent state $|\sqrt{\mu}e^{i\theta}\rangle$ of weak intensity. Since Eve and Bob have no information on θ , the state reduces to a mixed state $\rho = \int \frac{d\theta}{2\pi} |\sqrt{\mu}e^{i\theta}\rangle\langle\sqrt{\mu}e^{i\theta}|$ outside Alice’s laboratory. This state is equivalent to the mixture of Fock state $\sum_n p_n |n\rangle\langle n|$, with the number n of photons distributed as Possionian statistics $p_n = p_\mu[n] = \mu^n e^{-\mu} / n!$. The source that emits pulses in coherent states $|\sqrt{\mu}e^{i\theta}\rangle$ is equivalent to the representation as below: With probability p_0 , Alice does nothing; With probability p_n ($n > 0$), Alice encodes her bit in n photons. In order to gain Alice’s encoding information, Eve first performs a nondemolition measurement to gain the photon number of the laser pulses. When she finds there is only one photon in the pulses, she may implement symmetric individual (SI) attack on this qubit [12]. Otherwise, if there are two or more than two photons in the pulses, she may implement PNS attack on Alice’s qubit. In long distance QKD, the channel transmittance η can be rather small. If $\eta < (1 - e^{-\mu} - \mu e^{-\mu})/\mu$, Eve can gain full information of Bob’s final key by using the PNS attack [11].

In order to detect Eve’s PNS attack, Alice can introduce a decoy source μ' to ensure the security of their QKD. Since Bob’s detection apparatus is sensitive to the photon number, in the absence of Eve, photon number distributions in Bob’s detectors are also Poissonian

(Here, we assume that the dark counts rate r_{dark} in Bob's detectors is zero. We will discuss the realistic condition of that $r_{dark} > 0$ later.),

$$p_{sig}^{loss}[n] = \frac{(\eta\mu)^n}{n!} e^{(-\eta\mu)}, \quad (2)$$

$$p_{dec}^{loss}[n] = \frac{(\eta\mu')^n}{n!} e^{(-\eta\mu')} \quad (3)$$

Without the decoy state, the necessary condition of that Eve can implement her PNS attack without being detected is [11]

$$p_{sig}[n](1 - \sum_{i=0}^{n-1} f(n, i)) + \sum_{j=n+1}^{\infty} p_{sig}[j]f(j, n) \geq p_{sig}^{loss}[n], \quad (4)$$

where $f(m, k)$ is the probability of that Eve forwards k photons to Bob and stores the other $m - k$ photons. In general, let us assume Eve implements PNS attack P_n on Alice's pulses. Consider the case of that decoy states are used by Alice. Essentially, the idea of decoy-state is that [17]

$$P_n(signal) = P_n(decoy) = P_n \quad (5)$$

$$e_n(signal) = e_n(decoy) = e_n. \quad (6)$$

In this case, Eve can implement her PNS attack without being detected if and only if that

$$p_{sig}[n](1 - \sum_{i=0}^{n-1} f(n, i)) + \sum_{j=n+1}^{\infty} p_{sig}[j]f(j, n) = p_{sig}^{loss}[n], \quad (7)$$

$$p_{dec}[n](1 - \sum_{i=0}^{n-1} f(n, i)) + \sum_{j=n+1}^{\infty} p_{dec}[j]f(j, n) = p_{dec}^{loss}[n]. \quad (8)$$

Using the Taylor series, we can obtain that

$$f(n, i) = \binom{n}{i} \eta^i (1 - \eta)^{n-i}, \quad (9)$$

$$f(j, n) = \binom{j}{n} \eta^n (1 - \eta)^{j-n}. \quad (10)$$

Experimentally, these solutions just correspond to the case of that Eve blocks every photon with the probability $1 - \eta$, i.e., Eve forwards every photon with probability η through her lossless channel (This can be realized by using a beam splitter with the reflection probability $1 - \eta$ and the transmission probability η .). We will calculate the amount of information Eve can gain by using her PNS attack described by the equations (7) and (8) later.

III. COHERENT MULTIPHOTON PULSE ATTACK

From Eq.(1) we know that the rate of the secure final key is not only determined by the tagged counts but also determined by the QBER. That is, Eve may use some other eavesdropping schemes on the multiphoton pulses besides the PNS attack. Of course, these attacks will cause some QBER which could be detected in the verification test. A general attack scheme Eve may use is coherent multiphoton pulses attack. Let us first review the SI attack to introduce the CMP attack. When a photon propagates from Alice to Bob, Eve can let a system of her choice, called a probe, interact with the photon. Eve can freely choose probe and the initial state. But her interaction must obey the laws of quantum mechanics. That is, her interaction must be described by a unitary operator. After the interaction, Eve forwards the photon to Bob. Eve will perform a measurement on her probe to draw Alice's encoding information after Alice announces the basis she used. This is Eve's SI attack scheme. In the case of a multiphoton pulse, Eve will let her probes to interact with Alice's photons one-to-one. After Alice's announcements, Eve will perform a coherent measurement on her probes. We call this attack as CMP attack. Obviously, the simplest CMP attack is SI attack: If Alice sends a photon in the state $|\uparrow\rangle$, the result may be written as

$$U(|\uparrow\rangle|0\rangle) \rightarrow |X\rangle, \quad (11)$$

where $|X\rangle$ is the entangled state of the probe and the photon [25]. Likewise, we can obtain the state $|Y\rangle$, $|U\rangle$ and $|V\rangle$ corresponding $|\downarrow\rangle$, $|\rightarrow\rangle$ and $|\leftarrow\rangle$, respectively. In SI attack scheme, one can obtain that $|X\rangle = \sqrt{f}|\uparrow\rangle|\phi_{\uparrow}\rangle + \sqrt{e}|\downarrow\rangle|\theta_{\uparrow}\rangle$, $|Y\rangle = \sqrt{f}|\downarrow\rangle|\phi_{\downarrow}\rangle + \sqrt{e}|\uparrow\rangle|\theta_{\downarrow}\rangle$, $|U\rangle = \sqrt{f}|\rightarrow\rangle|\phi_{\rightarrow}\rangle + \sqrt{e}|\leftarrow\rangle|\theta_{\rightarrow}\rangle$ and $|V\rangle = \sqrt{f}|\leftarrow\rangle|\phi_{\leftarrow}\rangle + \sqrt{e}|\rightarrow\rangle|\theta_{\leftarrow}\rangle$, where f is the fidelity of the state and $f + e = 1$. From the unitarity of the interaction, we have that $\langle\phi_{\uparrow}|\theta_{\uparrow}\rangle = \langle\phi_{\downarrow}|\theta_{\downarrow}\rangle = \langle\phi_{\uparrow}|\theta_{\downarrow}\rangle = \langle\phi_{\downarrow}|\theta_{\uparrow}\rangle = 0$. It then follows from $\langle\phi_{\uparrow}|\phi_{\downarrow}\rangle = \cos\alpha$ that QBER=[1 - cos α]/2. The maximal information Eve can gain is that

$$I_{SI} = 1 - h\left(\frac{1 + 2\sqrt{e - e^2}}{2}\right), \quad (12)$$

where $h(x) = -x \log_2 x - (1 - x) \log_2(1 - x)$ and e is QBER.

In Eve's CMP attack scheme, she attaches her probes with all photons in the multiphoton pulse one-to-one. She interacts the probe-photon pair unitarily and then forwards the pulse

to Bob. She measures the probes coherently after Alice's announcements. This can be described as

$$[U(|\uparrow\rangle|0\rangle)]^{\otimes n} \rightarrow |X\rangle^{\otimes n}, \quad (13)$$

where $[U(|\uparrow\rangle|0\rangle)]^{\otimes n} = \underbrace{U(|\uparrow\rangle|0\rangle)\dots U(|\uparrow\rangle|0\rangle)}_n$, and $|X\rangle^{\otimes n} = \underbrace{|X\rangle\dots|X\rangle}_n$. Likewise, one can obtain $|Y\rangle^{\otimes n}$, $|U\rangle^{\otimes n}$ and $|V\rangle^{\otimes n}$. Suppose Alice announces that $|\uparrow\rangle, |\downarrow\rangle$ basis has been used. It has that

$$|X\rangle^{\otimes n} = (\sqrt{f}|\uparrow\rangle|\phi_{\uparrow}\rangle + \sqrt{e}|\downarrow\rangle|\theta_{\uparrow}\rangle)^{\otimes n}, \quad (14)$$

$$|Y\rangle^{\otimes n} = (\sqrt{f}|\downarrow\rangle|\phi_{\downarrow}\rangle + \sqrt{e}|\uparrow\rangle|\theta_{\downarrow}\rangle)^{\otimes n}. \quad (15)$$

Then the two density operators that Eve must distinguish are

$$\rho_{\uparrow} = \sum_{i=0}^n \frac{n!f^{n-i}e^i}{(n-i)!i!} |\phi_{\uparrow}\rangle^{\otimes n-i} |\theta_{\uparrow}\rangle^{\otimes i} (\langle\phi_{\uparrow}|)^{\otimes n-i} (\langle\theta_{\uparrow}|)^{\otimes i} \quad (16)$$

$$\rho_{\downarrow} = \sum_{i=0}^n \frac{n!f^{n-i}e^i}{(n-i)!i!} |\phi_{\downarrow}\rangle^{\otimes n-i} |\theta_{\downarrow}\rangle^{\otimes i} (\langle\phi_{\downarrow}|)^{\otimes n-i} (\langle\theta_{\downarrow}|)^{\otimes i} \quad (17)$$

The optimal information Eve can gain from these two states can be obtained as follow: Eve first performs the measurements on her probes. If her measurement results are that $|\phi_{\uparrow}\rangle^{\otimes n-i} |\theta_{\uparrow}\rangle^{\otimes i}$ (or $|\phi_{\downarrow}\rangle^{\otimes n-i} |\theta_{\downarrow}\rangle^{\otimes i}$), where $1 \leq i \leq n-1$, then Eve know that her density operator is ρ_{\uparrow} (or ρ_{\downarrow}) since $\langle\phi_{\uparrow}|\theta_{\uparrow}\rangle = \langle\phi_{\downarrow}|\theta_{\downarrow}\rangle = \langle\phi_{\uparrow}|\theta_{\downarrow}\rangle = \langle\phi_{\downarrow}|\theta_{\uparrow}\rangle = 0$. Only if the measurement results are $|\phi_{\uparrow}\rangle^{\otimes n}$, $|\theta_{\uparrow}\rangle^{\otimes n}$, $|\phi_{\downarrow}\rangle^{\otimes n}$, and $|\theta_{\downarrow}\rangle^{\otimes n}$, can Eve not distinguish her density operators. Suppose that Eve's measurement result is $|\phi_{\uparrow}\rangle^{\otimes n}$. From $\langle\phi_{\uparrow}|\phi_{\downarrow}\rangle = \cos\alpha$, we can obtain that

$$(\langle\phi_{\downarrow}|\phi_{\uparrow}\rangle)^{\otimes n} = \cos^n \alpha. \quad (18)$$

The maximal probability that Eve can distinguish ρ_{\uparrow} from ρ_{\downarrow} correctly is that $\frac{1+\sqrt{1-\cos^{2n}\alpha}}{2}$. Thus, the maximal information Eve can gain is that

$$\begin{aligned} I_{CMP}(n) &= (1-f^n-e^n) + f^n(1-h(\frac{1+\sqrt{1-\cos^{2n}\alpha}}{2})) + e^n(1-h(\frac{1+\sqrt{1-\cos^{2n}\alpha}}{2})) \\ &= 1 - (f^n+e^n)h(\frac{1+\sqrt{1-(1-2e)^{2n}}}{2}). \end{aligned} \quad (19)$$

That is, when Eve uses the CMP attack scheme, optimal information she can gain is $I_{CMP}(n)$. Suppose Eve interacts with n photons. If these n photons are from n independent qubits (Qubits are uncorrelated since weak coherent sources are used.), then information Eve can

gain is nI_{SI} . If these n photons are from a multiphoton pulse, then information Eve can gain is $I_{CMP}(n)$. When the QBER is small and the photon number n is not so big, we can gain that $I_{CMP}(n) \geq nI_{SI}$, see Fig.1. In fact, most of the multiphoton pulses are two-photon pulses since weak coherent sources are used experimentally. Numerical solution shows that $I_{CMP}(2) > 2I_{SI}$ if $e \leq 0.11$, at which error correction can be implemented. That is, CMP attack is more efficient than SI attack when weak coherent sources are used [26].

IV. FROM RAW KEY TO SIFTED KEY

From discussion above, we know that Eve can get more benefits from a multiphoton pulse than that from the single-photon pulse. Since Bob's detection apparatus can resolve the photon number of an arriving pulse, Alice and Bob can discard all of the multiphoton pulses out of the raw key to generate the sifted key. Therefore, only the pulses detected in Bob's detectors as the single photon pulses will be used to generate the sifted key. In this case, the fraction of counts caused by multiphoton pulses in the sifted key is that

$$\begin{aligned}\Delta &= \frac{\sum_{n=2}^{\infty} \mu^n e^{-\mu} \eta (1-\eta)^{n-1} n/n!}{\sum_{n=1}^{\infty} \mu^n e^{-\mu} \eta (1-\eta)^{n-1} n/n!} \\ &= 1 - e^{-\mu(1-\eta)},\end{aligned}\tag{20}$$

where

$$\lim_{\eta \rightarrow 0} \Delta = 1 - e^{-\mu}.\tag{21}$$

That is, the upper bound on the fraction of count caused by multiphoton pulses is $\Delta_0 = 1 - e^{-\mu}$ with high losses. This upper bound is approximate to μ when faint coherent sources are used. In order to gain the secure final key, a fraction $H_2(e)$ of the sifted key bits are sacrificed asymptotically to perform error correction and a fraction $H_2(e + \Delta_0)$ of the sifted key bits are sacrificed to perform privacy amplification [27]. After the correcting errors in the sifted key, Alice and Bob can execute privacy amplification in two different strings, the sifted key bits arising from the untagged qubits and the sifted key bits arising from the tagged qubits. The worst case assumption is that the bit error rate is zero for tagged qubits [7]. Therefore, secure final key can be extracted from sifted key at the asymptotic rate

$$R \geq (1 - \Delta_0) - H_2(e) - (1 - \Delta_0)H_2\left(\frac{e}{1 - \Delta_0}\right).\tag{22}$$

In the prior art GLLP [7], $\Delta_0 = p_{multi}/\mu$, where p_{multi} is the probability of Alice's emitting a multiphoton signal. This is the worst situation where all the multiphoton pulses emitted by Alice will be received by Bob. In our scheme, only "single photon" pulses detected in Bob's detectors are used to generate the sifted key. If this "single photon" pulse is a multiphoton pulse emitted from Alice, then we assume that it belongs to the tagged qubits. The other "single photon" pulses detected in Bob's detectors are real single photon pulses emitted from Alice. Thus, Eve's CMP attack can be ignored in our scheme.

V. PDS QKD WITH IMPERFECT PHOTON-NUMBER-RESOLVING DETECTORS

Resolving power of realistic photon-number-resolving detectors is finite. Suppose photon number resolving power of the detectors is n_0 . Let us assume that Eve can attack the photon pulses using PNS attack freely when the number of a pulses is bigger than n_0 . In this case, additional information Eve can gain is that

$$\Delta' = \frac{\sum_{n=n_0+1}^{\infty} \mu^n e^{-\mu} / n!}{\sum_{n=1}^{\infty} \mu^n e^{-\mu} \eta (1-\eta)^{n-1} / (n-1)!}. \quad (23)$$

Typically, $n_0 = 4$, $\eta = 10^{-3}$, $\mu = 0.1$. Then we can estimate that $\Delta' \lesssim 10^{-3}$, which is a very small quantity. The particular resolving power of detectors used in Ref.[24] can go up to 10 photons or so (~ 8 eV), so that the quantity $\Delta' \ll 10^{-10}$, which is negligible. In fact, Eve can not get benefit from the pulses $n > n_0$ since all of the multiphoton pulses detected in Bob's detectors are discarded, i.e., $\Delta' = 0$.

Another question is dark counts from blackbody photons propagating through the optical fiber. Fortunately, these photons can be filtered well. Experimentally, a really good filter (40 dB out-of-band rejection, 10nm wide passband), would result in 0.05 Hz of background counts [28]. Suppose the pulse rate emitted from Alice is r_{pul} and the dark count rate is r_{dark} Hz. We can obtain the *normalized* dark count rate d (dark counts per pulse) in Bob's detectors is that $d \approx \frac{r_{dark}}{r_{pul}\mu\eta}$. Distribution of dark counts in Bob's detectors is that

$$p_{dark}[n] = d^n. \quad (24)$$

Therefore, in experiment, Bob can obtain photon number distribution of the laser pulse by subtracting the dark counts from the real counts. QBER e_{dark} caused by dark counts should

be considered, especially in the long distance QKD,

$$e = e_0 + e_{dark}, \quad (25)$$

where $e_{dark} = d/2$, and e_0 is caused by the imperfections of the optical setup [1].

VI. OPTIMAL INTENSITY OF LASER SOURCE TO GENERATE SECURE FINAL KEY

In BB84, the rate of generating raw key is approximate to $\frac{1}{4}\mu\eta$. Thus, the rate of generating secure final key is approximate to $\frac{1}{4}\mu(1 - \Delta_0)\eta R$. That is, the rate of generating the secure final key is approximate to R_f , where

$$R_f = \frac{1}{4}\mu(1 - \Delta_0)\eta[(1 - \Delta_0 - H_2(e)) - (1 - \Delta_0)H_2(\frac{e}{1 - \Delta_0})], \quad (26)$$

where $\Delta_0 = 1 - e^{-\mu}$. In practice, e and η are constants when the transmission distance is constant. Therefore, the only variable in R_f is μ . R_f reaches its maximum at the point $\frac{\partial R_f}{\partial \mu} = 0$. In this way, we can obtain optimal parameter μ , see Fig. 2.

VII. DISCUSSION AND CONCLUSION

In the prior decoy state QKD [14,15,17], it requires that $\mu' > \mu$. In [14,15], the upper bound on the fraction of counts caused by the multiphoton is $\Delta \leq \frac{\mu e^{-\mu}}{\mu' e^{-\mu}}$. Only if $\mu = \mu'$ can the upper bound be reduced to μ [15]. In our scheme, μ is independent of μ' so that both signal pulse and decoy pulses can be used to generate the raw key. Another difference is that all the pulses detected in Bob's detectors are discarded in our scheme, so that Eve's CMP attack does not exist in our scheme. However, CMP attack should be considered in [14,15,17] to ensure the security of the final key.

In our scheme, from $\Delta = 1 - e^{-\mu(1-\eta)}$, we can conclude that the upper bound $\Delta_0 = 1 - e^{-\mu}$ can not be reduced any longer as long as weak coherent sources and high lossy channel are used, so that the quantity $\Delta_0 = 1 - e^{-\mu}$ is also the lower bound on the fraction of counts caused by the multiphoton pulses. Thus, the fraction $\Delta_0 = 1 - e^{-\mu}$ seems “*inherent*” in the long distance QKD with weak coherent sources and high lossy channel.

In summary, we have discussed the security of practical BB84 QKD protocol with weak coherent sources, noises and high losses. We have presented a PDS QKD scheme based

on recent experimental advancements. The upper bound on fraction of counts caused by multiphoton pulses is independent of the intensity of decoy source so that both the signal pulses and decoy pulses can be implemented to generate the raw key after verified the security of the QKD. We have shown that CMP attack is more efficient than SI attack. Finally, optimal μ is presented to improve the rate of generating the secure final key.

VIII. ACKNOWLEDGMENT

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- [27] P. Shor and J. Preskill, Phys. Rev. Lett. **85**, 441-444 (2000).
- [28] Maybe, we should assume that Eve can control the dark counts since Eve may change the wavelength of Alice's photon which is more sensitive for Bob's detectors. However, Bob can adds a filter in his laboratory to defeat Eve's such attacks. These days, the bandwidth of optical devices is as narrows as 0.1 to 0.01nm which is comparable to the laser linewidth. An optical grating to filter out unwanted frequencies may be used in combination with such the

narrow bandwidth devices; Experimental data were obtained from D. Rosenberg by private communication.

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X. CAPTION

Caption 1. (Color online.) Information vs photon number. Information Eve can gain from n photons by using SI attack (a) is nI_{SI} since these n photons come from n uncorrelated photon pulses. If these n photons are from a multiphoton pulse, then information Eve can gain is $I_{CMP}(n)$ (b). Numerical solution shows that $I_{CMP}(2) > 2I_{SI}$ when $e \leq 11\%$. And $I_{CMP}(3) > 3I_{SI}$ when $e \leq 6.8\%$. CMP attack is more efficient than SI attack since weak coherent sources are used experimentally.

Caption 2. (Color online.) Rate of generating final key vs transmission distance. In order to be comparable, we use the parameters in [17,29] instead of [24]. When $\mu = 0.1$, transmission distance is close to 140.2 km which is comparable with LMC in [17]. Numerical solution shows that optimal intensity of laser source is $\mu \approx 0.7$ (transmission distance over 164.1 km). That is, optimal intensity of laser source provides 23.9 km increase in the transmission distance. Transmission distance is stable to small perturbations to the optimal μ (up to 20% change of μ , less than 0.3% change of transmission distance). Here, we have verified that error correction are allowable for the maximal transmission distance.



